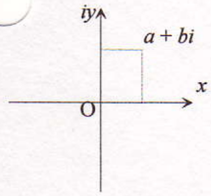
M1 Special Exercise

Complex Numbers



Law of Exponents:

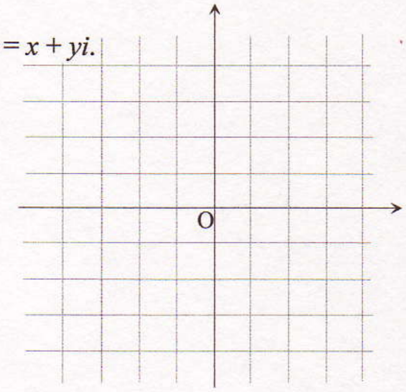
1. $a^n \equiv \overbrace{a \cdot a \cdot a \cdot \ldots \cdot a \cdot a}^{n \text{ objects}}$
2. $a^n \cdot a^m = a^{n+m}$
3. $(a^n)^m = a^{nm}$
4. $\dfrac{a^n}{a^m} = a^{n-m}$

   Especially,
   $a^0 = 1$
   $\dfrac{1}{a^m} = a^{-m}$
5. $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

$ax = b$
⇄

$ax^2 + bx + c = 0$
⇄

1. The complex number consists of the real part and the imaginary part: $z = x + yi$.
   Plot the following numbers in the complex plane.



(1) -2.5　(2) $-3-\sqrt{5}$　(3) $\sqrt[3]{-27}$　(4) $1+3i$

(5) $\dfrac{2}{3}+2i$　(6) $2+i\sqrt{3}$　(7) $-i\sqrt{7}$　(8) $\dfrac{-1}{i}$

2. Simplify.

(1) $\dfrac{3-\sqrt{5}}{2+\sqrt{5}}$　(2) $\dfrac{1+\sqrt{2}-\sqrt{3}}{1+\sqrt{2}+\sqrt{3}}$　(3) $(1+i)^2$　(4) $(2-i\sqrt{3})^3$

(5) $\sqrt{14-2\sqrt{33}}$　(6) $\sqrt{6-3\sqrt{3}}$　(7) $\sqrt{\dfrac{2-\sqrt{3}}{2+\sqrt{3}}}+\sqrt{\dfrac{2+\sqrt{3}}{2-\sqrt{3}}}$　(8) $\left(\dfrac{1+i}{\sqrt{2}}\right)^8$

3. Express in terms of exponents.

(1) $a^3 \times a^{-2}$　(2) $a^{-6} \times a^3 \times a^{-2}$　(3) $a^{-3} \div a^{-2} \div a^4$　(4) $(a^{-4})^{-1}$　(5) $(ab^{-2})^3$

(6) $(a^{-2}b)^{-2}$　(7) $(a^{-1}b^{-2}c^3)^{-2}$　(8) $(a^{-1}b)^2 \times (ab^{-1})^{-1}$　(9) $a^2bc^3 \div (-a^2bc)^3 \times (ab^{-1}c^2)^{-3}$

4. Simplify.

(1) $\sqrt[4]{64} \div \sqrt[4]{4}$　(2) $\sqrt[3]{24} \times 3\sqrt[3]{3} \times \sqrt[3]{81}$　(3) $\sqrt[3]{\sqrt{216}} \times \sqrt{36} \div \sqrt[3]{-27}$

(4) $\sqrt{\sqrt[3]{a^6}}$　(5) $\sqrt{a} \times \sqrt[3]{a} \times \sqrt[6]{a}$　(6) $32^{\frac{2}{5}}$

(7) $2^{\frac{5}{3}} \times 2^{\frac{3}{2}} \div 2^{\frac{7}{6}}$　(8) $3^{\frac{1}{3}} \times 3^{\frac{3}{2}} \div 3^{\frac{9}{4}}$　(9) $\left\{\left(\dfrac{9}{16}\right)^{-\frac{3}{4}}\right\}^{\frac{2}{3}}$

(10) $(x-y^{-1}) \div (x^{\frac{1}{2}} - y^{-\frac{1}{2}})$　(11) $(a^{\frac{1}{4}} - b^{\frac{1}{4}})(a^{\frac{1}{4}} + b^{\frac{1}{4}})(a^{\frac{1}{2}} + b^{\frac{1}{2}})$

5. Solve the following equations.

(1) $ax = b-1$　(2) $a^2(x+1) = a(x+3)-2$　(3) $(1+2\sqrt{5})x^2 - 2(4+\sqrt{5})x + 7 = 0$

6. Define the operator * by $a*b \equiv a+b-ab$, where both a and b belong to the set of rational numbers $\mathbb{Q}$.

(1) Calculate (i) $\dfrac{1}{2}*\dfrac{1}{3}$　(ii) $\left(\dfrac{1}{2}*\dfrac{1}{3}\right)*\dfrac{1}{4}$

(2) Let the set of integers be $\mathbb{I}$. Show that the set of integers $\mathbb{I}$ is closed in terms of the operator *.

(3) Examine whether the commutative law holds or not for the set of integers $\mathbb{I}$. Does the associative law hold for $\mathbb{I}$?