Sequence Quick Review

Formula Summary

❑Arithmetic Sequences (or Progressions)

$a_n - a_{n-1} = d \rightleftarrows a_n = a_1 + (n-1)d$

Sum of the First n Terms: $S_n = \frac{n(a_1 + a_n)}{2} = \frac{n\{2a_1 + (n-1)d\}}{2}$

❑Geometric Sequences (or Progressions)

$a_n = ra_{n-1} \rightleftarrows a_n = a_1 r^{n-1}$

Sum of the First n Terms: $S_n = a_1 + a_1 r + a_1 r^2 + \ldots$

To derive the formula, multiply by r, then subtract.

$$\begin{cases} \text{If } r \neq 1, & = \dfrac{a_1(1-r^n)}{1-r} \\ \text{If } r = 1, & = na_1 \end{cases}$$

❑Sigma Notation and Sum of the Natural Numbers in Power: k, k^2, k^3...

To derive the formula, use the identities such as:

$(k+1)^2 - k^2 = 2k + 1$ for Σk

$(k+1)^3 - k^3 = 3k^2 + 3k + 1$ for Σk^2

$(k+1)^4 - k^4 = 4k^3 + 6k^2 + 4k + 1$ for Σk^3

$\Sigma k = \frac{n(n+1)}{2}$

$\Sigma k^2 = \frac{n(n+1)}{2} \cdot \frac{(2n+1)}{3}$

$\Sigma k^3 = \left(\frac{n(n+1)}{2}\right)^2$

$\Sigma k^4 = \frac{n(n+1)}{2} \cdot \frac{(2n+1)}{3} \cdot \frac{(3n^2 + 3n - 1)}{5}$

$\Sigma k^5 = \left(\frac{n(n+1)}{2}\right)^2 \cdot \frac{(2n^2 + 2n - 1)}{3}$

❑ Recursive Form of Sequences

1) Relations between two successive terms

$a_n = ra_{n-1} + d$

$$\begin{cases} \text{If } r = 1, \ a_n - a_{n-1} = d \\ \text{If } d = 0, \ a_n = ra_{n-1} \end{cases}$$

2) Relations in Three successive terms

$a_{n+1} + pa_n + qa_{n-1} = r$

3) Relations in Rational Forms

$a_n = \frac{pa_{n-1} - q}{a_{n-1} - r}$

❑Mathematical Induction

1. First, prove that the given expression holds for $n = 1$. (or any initial value)
2. Assume that the given expression holds for n.
3. If you derive a new expression that holds for $n + 1$,
   then the given expression is to hold for all range of natural numbers n.

❑Various Sequences

Find some general rules that may be derived from the given sequences.
Then, use the typical sequences that are already familiar to you.